Modelling the Uncertainty in the Peruvian Stock and Forex markets with Dynamic Conditional Score Models*

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*I acknowledge that this is my own work, and I have read and understood WKH Section on plagiarism.**

Abstract

I study a Dynamic Conditional Score model for modelling volatilities, or the uncertainty, (Beta-t-EGARCH) with Random Level Shifts (RLS) following the work of Qu and Perron [*Econometrics Journal* (2013) vol. 16, pp. 309–339] in financial markets. The addition of random level shifts can explain the high persistence typically estimated for these series, this constitutes an alternative approach to the long memory or two component models. I also model the asymmetries between returns and volatility within this framework. Hence, I model the uncertainty in the Peruvian Stock and Forex Peruvian market using daily data from the last two decades. The estimates of the RLS component and volatilities fits well the main disturbance events in the period of study. I study how the volatilities of both markets match a model of short memory plus RLS. Finally, I carried out Monte Carlo simulations which shows how accurate is the model proposed, since is able to follow the time and spectral domain properties of the original series.

Contents

1 Introduction

A recent observation driven approach for modelling financial series denominated Dynamic Conditional Score (DCS) models of Harvey (2013) or also known as Generalized Autoregressive Models as in Creat et al. (2011, 2013) is emerging as an alternative to the traditional Generalized Autoregressive Conditional Heteroskedasticity (GARCH) and Stochastic Volatility (SV) models. According to Harvey (2013), this model offer a flexible structure as it gives a unified framework to modelling time varying parameters. For example, the location or the scale of a heavy tailed distributions, or for modelling time varying correlation, dynamic copulas proposed in Oh and Patton (2017), the modelling of time between trades in stock markets with duration models. Also, we can find current contributions for the study of censored distributions as in Harvey and Ito (2017). They study time series with a considerable amount of zeroes in the data.

In particular, little work has been made in the study of why DCS models estimates a high persistent scale/volatility. Harvey (2013), for instance, consider a two component model which aims to capture somewhat the persistence in the volatility using jointly a long memory and the short memory process. In this dissertation, I study a new approach in order to model the persistence of volatilities (uncertainty) within a DCS framework, typically displayed in high frequency data. To do so, I follow the idea of Qu and Perron (2013) (QP) that random level shifts and a short memory component can reproduce the main features of these series. Hence, I modify the Beta-t-EGARCH model for volatilities of Harvey (2013).

This observation driven model does not suffer from the nonexistence of moments as the EGARCH model of Nelson (1991) and at the same time this model allows modelling directly the volatility through the scale of the t-Student distribution. To my best of knowledge, this is the first work to address the high persistence of series with random level shift (RLS) in a DCS setting. Others related models consider change in regimen using a Markov Switching model as in Bazzi et al. (2017) and Blazsek and Ho (2017). However, both models are limited to study of up to two regimes. In this dissertation multiple regimes are modelled for the Peruvian Stock and Forex markets volatilities.

There are numerous studies exploring the causality relationship between economic growth and the dynamic of stock markets. Enisan and Olufisayo (2009) find a positive relationship for a group of emerging countries. Levine (2006) argues that investors who buy and sell liquid instruments in this market trigger the transmission channel from financial to real sectors. Though inaugurated at the beginnings of 1990s, the Lima stock market have growth steeply. For instance, by 2016 the stock market listed 283 of the main companies of the country and had a stock valuation of around 125 000 millions USD, which meant 66 per cent of the gross domestic product at that age.

Likewise, the understanding about the dynamics of the forex market is crucial in a dollarized economy as the Peruvian. Grippa and Gondo (2006) explains how high uncertainty in the forex market can damage the financial conditions of individuals who owns debt or take loans in the foreign currency. According the Inflation report of the Central of Peru in 2016 the dollarization of credits and in the liquidity reached levels of 29 percent and 49 percent respectively. No much study about these Peruvian financial markets has been made until the work of Humala and Rodriguez (2013). A more recent study is made in Alvaro et al. (2017). They apply the QP model to six commodities prices that highlight the importance of shifts in the modelling of uncertainty. The main contribution of this dissertation is the incorporation of random level shifts a la QP into the Beta-t-EGARCH model. Using the SV specification of Qu and Perron (2013) I estimate the multiple regimes that governs the uncertainty in both markets, then I add these estimates to the DCS model of Harvey (2013) to finally estimate the whole model in a two-step procedure.

I find that the persistence of a shock in a model without RLS for both financial markets has a half-life of around 90 days. Nonetheless, if we model for RLS within the Beta-t-EGARCH model the effects of shocks last a few more that 10 days each. This results show the importance of modelling multiple regimes in the volatility because this will have a significant effect in the forecast of the duration of shocks, and hence, in the transitory effects of a shock to the volatility. The regimes detected coincide with periods of very high uncertainty mainly from the internal election which happens around the national elections where the left candidate Humala exchaerbit the investment behavior with the named "Peruvian Black Monday" where the index lost 12 percent its value, thus with immediate consequences not only in the stock market but in the forex market, due to the outflows of speculative international investors. Likewise, another big event which affected financial markets globally was the financial crisis in 2008, having a strong impact to the economy which that year did not growth after 10 years.

Interestingly, for the forex market after those great crashes the upward level shifts it revert within few months to levels before the crash. This is due to the intervention of the Central Bank of Peru which has a floating regime system in which they aims not to directly control the volatility but they have as a policy smooth the path of the volatility against times of high uncertainty. In this line in order to study the different reactions of the dynamics of the volatility from shocks in its process I modify the model to include asymmetries between returns and the volatility. For the stock market the results reveals a positive sign for the leverage parameter. In other words, a negative shock to the stock market originating more volatility than a positive shocK. Alike, for the forex volatility dynamics I found a negative elasticity. Positive shocks in this market are conceived as currency depreciation shocks and therefore, in a partially dollarized economy, it have a big impact in the uncertainty for people who owns credit in this money, but receives earning in the local currency. As a result, a negative coefficient for the asymmetries means that depreciation effects rather than appreciation generates more uncertainty in the Forex market. Thus, the Central Bank should be relatively aware when the local currency lose its value and try to mitigate the uncertainty in this market.

Further, I find from the log periodogram of Perron and Qu (2010) that both financial series present patterns which match a process for short memory in volatility and level shifts. As argued in Perron and Qu (2010) the rapid decay in the spectral log periodogram at higher frequencies is a proper characteristic of series with short memory and random level shifts. Hence, other models such as of the fractional integration or a two component models that aim to estimate directly the long memory parameter may not be appropriate. Monte Carlo simulations show that the model proposed here is able to match the time and frequency properties of the empirical series, due they assume per se the existence of are not of the best and those results are corroborated

2 Literature Review

One of the main contribution in the study of time varying volatilities is the Autoregressive Conditional Heteroscedasticity (ARCH) model of Engle (1982), which he uses to study the UK inflation volatility. One main drawback of this model comes from the highly persistent estimated volatility, as a result, numerous lags are necessary to control it, and this may imply the estimation of negative parameters and the requirement to impose additional restrictions. Bollerslev (1986) overcome these limitations and propose the Generalized Autoregressive Heteroscedasticity (GARCH) model. This model can group the extensive lags of the ARCH model by adding as an explanatory variable for the volatility, a lag term of it.

On the other hand, models that assume an unobservable process for the volatility is the family of Stochastic Volatility (SV) models. In its discrete version, Jaquier et al. (1994) analyses different approaches to estimate the SV model, he found that the Bayesian estimation gains in efficiency with respect to the Quasi maximum likelihood of Harvey et al. (1994), who uses a Kalman filter methodology, since it is a relatively more efficient algorithm for the convergence of the parameters to their true values.

Nonetheless, there is a recent approach from the Dynamic Conditional Score theory of Harvey (2013) and Creal et al. (2011, 2013)[1]. The main advantages from this view is that it encompasses the GARCH in a more flexible setting, at the same time it surpass their major limitations such as the restriction in the parameters of models. Harvey (2013) highlights the robustness of the model to outliers or atypical observations when a t-Student version of the model is taken into account.

A concern in the literature of financial series is the high persistence of the conditional or latent volatilities when estimated, some studies treat this phenomenon as series with long memory. This series are characterized by a hyperbolic decay in their autocorrelation functions, in other words, a shock has a long persistence. Using proxies of volatilities such as the absolute value of the returns, Hosking (1980) and Granger and Joyeux (1980) propose the fractional integration model, the ARFIMA(p, d, q) and $d > 0$ implies that a long memory is present in the series. In this line, Harvey (1998) and Breidt et al. (1998) model the fractional integral

[1] In this article the model is named Generalized Autoregressive Score (GAS).

parameter d in a SV framework. For DCS models, Harvey (2013) performs a two component model, where one process is of long memory and the other the short one.

Nonetheless, instead of model directly fractional integration, Diabold and Inoue (2001) show how a long memory process can be confound with a model that incorporates a structural break. This idea is related to Perron (1989) where breaks in a unit root setting can conduct to misleading results when the test is performed. Likewise, Starika and Granger (2005) shows that level shifts account significantly in the dynamics of the unconditional variance of S&P 500. Perron and Qu (2010), who also analyses this index, they carry out Monte Carlo simulations to show how the short memory process and level shifts affects the estimation of d and that this mixture model mimics the main patterns such as the ACF and the periodogram of these series.

How the volatility overreact from good news is studied by Nelson (1991) who propose the model called exponential GARCH (EGARCH); given its exponential nature, this specification does not arise problem with non-negative values for the coefficients which could mean negative values for the volatility, if not adequate restrictions are in place for the parameters. A similar work is made by Glosten et al. (1993) and its model GJR-GARCH also studies the asymmetries generated from unexpected positive or negative shocks over the following period conditional volatility. On the side of SV models, the extension is made by Yu (2005) and Omori et al. (2007), they incorporate asymmetries in the dynamics of the volatility for model the S&P 500 and Asian stock markets. They developed Bayesian algorithms in the search of gain in the efficient of the algorithm. Given its adaptability, the DCS model can be easily modified to include leverage as in Harvey (2013) where he apply this model to the indexes Hang Seng of Hong Kond and the Down Jones.

As mentioned before, little work on the study of financial market has been developed. A first study analyzing the main stylized facts of the Peruvian Markets is Humala and Rodríguez (2013). In addition, competitive GARCH models are fitted to these series in Rodríguez (2017) where several univariate specifications for the conditional variance are compared and described. There is some literature of applied SV models to those returns as in the work of Alanya and Rodriguez (2014) where through several statistics, they compare its performance relative to a Normal and t-Student GARCH model, showing the gains in fit of the former model. Recently, Alvaro et al (2017) apply the RLS model of Qu and Perron (2013) to six commodities returns. They find that the RLS process contributes significantly to the volatility process. In addition, they study their correlation with the main economic activity index and found positive correlations with the business cycles of the Peruvian economy.

The structure followed by this paper is as follows: Section 3 discusses the Beta-t-EGARCH model with random level shifts and the procedure for its estimation. Section 4 presents the main empirical findings in the application of the model for the volatility to the Peruvian Stock and exchange rate returns. Section 5 shows through Monte Carlo simulations the relevance of random level shifts as a new approach to understand the high persistence in Dynamic Conditional Score models. The conclusions are set out in the final section.

3 Methodology

This section presents the SV and DCS models for measuring the uncertainty. In the same manner, this section introduces the Beta-t-EGARCH with random level shifts. Here, it is also explained the main algorithms in order to estimate these financial econometrics models.

3.1 The SV framework

Stochastic volatility models assume that volatilities follow a latent stochastic process. An advantage of these kind of models against the GARCH models is that SV models assumes an independent process to the equation for volatilities, in contrast to the GARCH family models where an unique innovation drives both the returns and volatility. QP incorporate a random level shifts process to the canonical SV in order to model the high persistence of financial series. The QP SV model for a demeaned series y_t has the following representation:

$$
\begin{aligned}
y_t &= \exp(h_t/2 + \mu_t/2)\varepsilon_t, \qquad (1) \\
h_{t+1} &= \phi h_t + \sigma_\upsilon \upsilon_t, \\
\mu_{t+1} &= \mu_t + \sigma_\eta \delta_t \eta_t. \\
\delta_t &\sim B(1,\, p), \\
\varepsilon_t &\sim N(0,1), \\
\upsilon_t &\sim N(0,1), \\
\eta_t &\sim N(0,1),
\end{aligned}
$$

where ε_t, υ_t, η_t and δ_t are mutually independent distributed processes. h_t is the first order stochastic volatility at time t and σ_υ its variance. The random level shifts process, μ_t, follow a similar structure to the local level (LL) model and so it is stochastic in nature, which is regulated by the innovation η_t, but a main difference with LL is that RLS incorporate random jumps given by a Bernoulli sequence δ_t which takes the value of 1 whit a probability of p and zero otherwise. In addition, σ_η reflects how much strong is the impact of a level shift in this stochastic process. The starting value for both processes h_0 and μ_0 are zero, meanwhile for their first observations is assumed an a priori distribution such that $(h_1, \mu_1)' \sim N(0, P)$. According to Qu and Perron (2013), this time varying parameter model differs from long memory models, because the short memory process will have a transitory effect in the volatility, whilst the level shifts captures the permanent effect until a new change in the regime occurs.

Stochastic volatility models allows for a linear version which ease its estimation. Hence, I take logarithms to the squares to the model in 1:

$$
\begin{aligned}
y_t^2 &= \left[\exp(h_t/2) + \exp(\mu_t/2)\right]^2 (\varepsilon_t)^2, \qquad (2) \\
\log(y_t^2) &= \mu_t + h_t + \log(\varepsilon_t^2), \\
h_{t+1} &= \phi h_t + \sigma_\upsilon \upsilon_t, \\
\mu_{t+1} &= \mu_t + \sigma_\eta \delta_t \eta_t.
\end{aligned}
$$

A problem with this specification comes from the error term, $\log(\varepsilon_t^2)$, since if it is assumed that a Gaussian distribution for ε_t, it will follow a χ^2 distribution. Thus, if a state space system is estimated by quasi maximum likelihood as in Harvey et al. (1994) assuming Gaussianity, this would lead to unbiased estimates, especially when a short number of observations is used. Another approach is adopted in Kim *et al.* (1998) who approximates ε_t^* using a mixture of normal distributions, they show that a mixture of 7 normal distributions $\sum_{i=1}^{k} q_i N(m_i, \sigma_i^2)$ with mean m_i, variance σ_i^2, and associated weights q_i generate a closer fit to the density of $\log(\varepsilon_t^2)$. I report the values for the mixture of normals in Table 2. Hence, Qu and Perron (2013) define a new zero mean error term, $\varepsilon_t^* = \log(\varepsilon_t^2) - E[\log(\varepsilon_t^2)]$, and the system becomes:

$$
\begin{aligned}
\log(y_t^2 + c) - E[\log(\varepsilon_t^2)] &= \mu_t + h_t + \varepsilon_t^*, \\
z_t &= \mu_t + h_t + \varepsilon_t^*, \\
h_{t+1} &= \phi h_t + \sigma_\upsilon \upsilon_t, \\
\mu_{t+1} &= \mu_t + \sigma_\eta \delta_t \eta_t.
\end{aligned}
$$

Note that it has been introduced the Fuller (1996) offset value $c = 0.001$ for the returns z_t in order to avoid values of the logarithm near to zero which may have an effect over the estimates of the model. Kim *et al.* (1998) defines $\omega_t = j$ if the realization from the mixture comes from its jth component. This will allow for the state space representation of the system.

3.1.1 Priors and Sampling for the Posterior Distributions

Stochastic volatility models are parameter driven models and so it requires th estimation of latent variables $\alpha_t = (h_t, \mu_t)'$. Firstly, it will be required the estimation of the parameter of the model $\theta = (\phi, \sigma_\upsilon, \sigma_\eta, p)$ using Markov Chain Monte Carlo (MCMC) Bayesian algorithms from which is possible to approximate the likelihood function $f(y|\theta) = \int f(y|\alpha, \theta) f(\alpha|\theta) dh$ of the model. A closed form for the likelihood does not exist as it entails unobservable variables, as argued in Jaquier et al. (1994).

In the Bayesian formulation in order to obtain posterior distributions of the parameters it is required the use of some believes, or priors, distributions according to the Bayes rule. A summary of the prior distributions employed and the implied prior means distributions are shown in Table 3. In particular, it is settled from the prior distribution that a shift happens each 41 days which given the size of the sample would imply around 100 shifts in the whole period. Secondly, using particles filters algorithms filtered and smooth estimates are obtained for the latent variables of the model.

Hence, the sampling procedure of Qu and Perron (2013) which follows a Gibbs Sampler algorithm is:

1. Initialize $\alpha_1 = (h_1, \mu_1)'$, $\theta = (\phi, \sigma_\upsilon, \sigma_\eta, p)$, $R = [(\upsilon_1, \eta_1)', ..., (\upsilon_n, \eta_n)']$, $\delta = (\delta_1, ..., \delta_n)$, $\omega = (\omega_1, ..., \omega_n)$.

2. Sample $\theta_{(-p)}$ and the latent process R from the joint posterior distribution $f(\theta_{(-p)}, R, \alpha_1|p, \delta, \omega, y) = f(R, \alpha_1|\theta, \delta, \omega, y)f(\theta_{(-p)}|p, \delta, \omega, y)$. The draws for $f(R, \alpha_1|\theta, \delta, \omega, y)$ are obtained from the De Jong and Shephard (1995) simulation smoother. Further, the density $f(\theta_{(-p)}|p, \delta, \omega, y)$ is approximated iteratively from the Gibbs Sampler for each of the parameters in $\theta = (\phi, \sigma_\upsilon, \sigma_\eta, p)$. For instance, for the parameter σ_υ and using the Bayes theorem: $f(\sigma_\upsilon|p, \delta, \omega, y) = f(\sigma_\upsilon|p, \delta, \omega, y)\pi(\sigma_\upsilon)$. Through the Kalman filter samples for $f(\sigma_\upsilon|p, \delta, \omega, y)$ are generated; on the other hand $\pi(\sigma_\upsilon)$ is the prior distribution for σ_υ. Then, both draws are combined employing the Gilks et al. (1995) Metropolis Sampler.

3. Sample the shifts δ_t from $f(\delta_t|\theta, \delta_{(-t)}, R, \alpha_1, \omega, y)$. Samples are computed iteratively for $t = n, n-1, ..., 1$ from the odds ratio, since it is computationally more efficient:

$$\frac{f(\delta_t = 1|\theta, \delta_{(-t)}, R, \alpha_1, \omega, y)}{f(\delta_t = 0|\theta, \delta_{(-t)}, R, \alpha_1, \omega, y)} = \frac{p \prod_{j=t+1}^{n} f(y_j|\theta, \delta_t = 1, \delta_{(-t)}, R, \alpha_1, \omega, Y_{j-1})}{(1-p) \prod_{j=t+1}^{n} f(y_j|\theta, \delta_t = 0, \delta_{(-t)}, R, \alpha_1, \omega, Y_{j-1})},$$

   where $Y_t = (y_1, ..., y_t)$.

4. Sample the shifts probability p using $f(p|\theta_{(-p)}, \delta, R, \alpha_1, \omega, y) = f(p|\delta) \propto f(p|\delta)f(p)$.

5. Sample the mixture ω with $f(\omega_t = j|\theta, \delta, R, \alpha_1, y) = f(\omega_t = j|\varepsilon_t^*) \propto f(\varepsilon_t^*|\omega_t = j)f(\omega_t = j)$ where the marginal distribution for ε_t^* is given by $\varepsilon_t^*|\omega_t = j \sim N(m_j, \sigma_j^2)$. In addition, QP suggest the reweighting procedure of Kim et al. (1998) to deal with numerical approximations in the mixture sample step within the algorithm.

3.1.2 Filtering for Latent Variables

The second block of variables for estimation in the SV model with random level shifts involves the filtering of latent processes for $\alpha_t = (h_t, \mu_t)'$. The filtering step will also allow for the approximation of the log likelihood of the model and is the main input for one or more step ahead forecasts. Qu and Perron (2013) adopted the particle filters of Gordon et al. (1993) and Kim et al. (1998) in order to approximate:

$$f(\alpha_{t+1}|Y_{t+1}, \theta) \propto f(y_{t+1}|\alpha_{t+1}, Z_t, \theta) \int f(\alpha_{t+1}|\alpha_t, Y_t, \theta) dP f(\alpha_t|Y_t, \theta).$$

To do so, QP generate $j = 1, ..., M$ samples α_t^j using $(\alpha_t|Y_{t,}\theta)$. Then, the draws for $f(\alpha_{t+1}|Y_{t+1}, \theta)$ come from reweighting $f(\alpha_{t+1}|\alpha_t^j, Y_t, \theta)$ with ω_{t+1}^j, where:

$$\begin{aligned} \omega_{t+1}^j &= \frac{f(y_{t+1}|\alpha_{t+1}^j, Y_t, \theta)}{\sum_{j=1}^M f(y_{t+1}|\alpha_{t+1}^j, Y_t, \theta)}, \\ f(y_{t+1}|\alpha_{t+1}^j, Y_t, \theta) &\sim N(0, \exp(h_{t+1}^j + \mu_{t+1}^j)). \end{aligned}$$

In addition, $f(\alpha_{t+1}|\alpha_t^j, Y_t, \theta)$ is a function of the probability that a level shift happens at t, so that:

$$\begin{aligned} f(\alpha_{t+1}|\alpha_t^j, Y_t, \theta) &\sim \delta_t W_{1t}^j + (1 - \delta_t) W_{2t}^j, \\ W_{1t}^j &\sim N\left(\begin{bmatrix} \phi & 0 \\ 0 & 1 \end{bmatrix} \alpha_t^j, \begin{bmatrix} \sigma_\upsilon^2 & 0 \\ 0 & \sigma_\eta^2 \end{bmatrix} \right), \\ W_{2t}^j &\sim N\left(\begin{bmatrix} \phi & 0 \\ 0 & 1 \end{bmatrix} \alpha_t^j, \begin{bmatrix} \sigma_\upsilon^2 & 0 \\ 0 & 0 \end{bmatrix} \right). \end{aligned}$$

3.2 A Beta-t-EGARCH model with Random Level Shifts

This section introduces the foundation models for the RLS-Beta-t-EGARCH model I propose in this dissertation. Hence I explain how these models are related to other financial econometrics settings such as GARCH, and the relevance for the use of DCS models which overcome limitations in the modelling of the uncertainty of these models.

3.2.1 GARCH and t-GARCH

The first order GARCH model of Taylor (1986) and Bollerslev (1986) for the conditional variance ${\sigma_{t|t-1}}^2$ of demeaned returns y_t is:

$$\begin{aligned} y_t &= \sigma_{t|t-1}\varepsilon_t, \qquad (3) \\ \varepsilon_t &\sim N(0,1), \\ \sigma^2_{t+1|t} &= \delta + \beta\sigma^2_{t|t-1} + \alpha y_t^2, \end{aligned}$$

with β and α are imposed to be non-negatives and δ strictly positive. Note that this model is a linear function of the lag of the conditional volatilities and also function of past squared returns. Consider now the model in terms of its persistence $\phi = \alpha + \beta$:

$$\begin{aligned} \sigma^2_{t+1|t} &= \delta + \phi\sigma^2_{t|t-1} + \alpha[y_t^2 - \sigma^2_{t|t-1}], \\ \sigma^2_{t+1|t} &= \delta + \phi\sigma^2_{t|t-1} + \alpha\upsilon_t, \end{aligned}$$

where υ_t is a martingale difference (MD). Harvey (2013) highlights the importance of this specification, in terms of a MD, which allows a link of the GARCH model with the signal extraction principle of the Kalman filter that is related to the DCS approach for modelling volatilities.

[2] Andersen et al. (2006) adopt this notation instead of σ_t since it depends on up $t-1$ observations and the estimation procedure involves filtering procedures.

3.2.2 Beta-t-GARCH

To examine more closely the relationship between GARCH and DCS models, let us consider now the Beta-t-GARCH[3] model of Creal et al. (2011, 2013) and Harvey (2013):

$$
\begin{aligned}
y_t &= \sigma_{t|t-1}\varepsilon_t, \\
\sigma^2_{t+1|t} &= \gamma + \phi\sigma^2_{t|t-1} + \theta\sigma^2_{t|t-1}u_t, \\
u_t &= \frac{(\nu+1)y_t^2}{(\nu-2)\sigma^2_{t|t-1} + y_t^2} - 1, \\
u_t &\sim beta(1/2, \nu/2), \\
\nu &> 2.
\end{aligned}
\tag{4}
$$

The conditional variance has persistence ϕ and depends on the martingale difference component u_t, through the elasticity with respect to the conditional volatility, θ. Further, the term u_t follows a Beta distribution[4] and this process is proportional to the score of volatilities, $\sigma^2_{t|t-1}$. To examine how is derived the score, let us consider the t-Student distribution with scale $\sigma^2_{t|t-1}$ and a location of zero has density:

$$
f(y;\sigma^2_{t|t-1}) = \frac{\Gamma((\nu+1)/2)}{\Gamma(\nu/2)\sigma^2_{t|t-1}\sqrt{\pi(\nu-2)}}\left(1 + \frac{y_t^2}{(\nu-2)\sigma^2_{t|t-1}}\right)^{-\frac{\nu+1}{2}},
$$

Thus, the log-likelihood for the observation y_t is:

$$
\begin{aligned}
\log f_Y(y_t) \;=\;& \log\left(\Gamma(\frac{\nu+1}{2})\right) - \log\left(\Gamma(\frac{\nu}{2})\right) - \frac{1}{2}\log((\nu-2)\pi\sigma^2_{t|t-1}) \\
&-\frac{\nu+1}{2}\log\left(1 + \frac{y_t^2}{(\nu-2)\sigma^2_{t|t-1}}\right).
\end{aligned}
$$

Hence, the first derivative of this expression with respect to $\sigma^2_{t|t-1}$ will give the

[3]As appointed in Ito (2016), the Beta-t-GARCH model is a DCS model where a square root link function for the volatility and an assumed centered standard t-Student distribution drives the volatility process.

[4]This is why the model is called Beta-t-GARCH.

score s_t and u_t being proportional to it:

$$
\begin{aligned}
s_t &= \frac{\partial \log f_Y(y_t)}{\partial \sigma^2_{t|t-1}} = \frac{\sigma^2_{t|t-1}}{2}\left(\frac{(\nu+1)y_t^2}{(\nu-2)\sigma^2_{t|t-1}+y_t^2}-1\right), \\
s_t &= \frac{\sigma^2_{t|t-1}}{2}u_t.
\end{aligned}
$$

The idea of Harvey and Chakravarty (2009) and Harvey (2013) to model the volatility using the score comes because it performs a better estimator for the conditional variance when the degrees of freedom ν is finite as it would lead to a non efficient estimator for the variance. In contrast, when ν goes to infinite, the system collapse to a standard GARCH model of Bollerslev (1986).

3.2.3 Beta-t-EGARCH

The central difference from the Beta-t-GARCH is that the Beta-t-EGARCH model uses an exponential function in order to model the scale of the t-Student distribution. This link exponential function allows that volatilities always take non negative values with the no necessity of impose any restrictions to its dynamics. Although Harvey (2013) describe a representation where there is an equivalence between the Beta-t-EGARCH model and the t-GARCH specification. The model is given by

$$
\begin{aligned}
y_t &= \exp(\lambda_{t|t-1})\varepsilon_t, \qquad (5)\\
\lambda_{t+1|t} &= \omega(1-\phi)+\phi\lambda_{t|t-1}+\kappa u_t, \\
u_t &= \frac{(v+1)y_t^2}{v\exp(2\lambda_{t|t-1})+y_t^2}-1. \\
\varepsilon_t &\sim t(v)
\end{aligned}
$$

where ω is the unconditional mean for the process of scale/volatility, and u_t is proportional to the score as in the Beta-t-GARCH. We should bear in mind that the scale and volatility are directly in this model, so that:

$$
\begin{aligned}
\varphi_{t+1|t} &= (v-2)^{1/2}\sigma_{t+1|t}, \\
\lambda_{t+1|t} &= \log(\varphi_{t+1|t}).
\end{aligned}
$$

3.2.4 RLS-Beta-t-EGARCH

This specification combines the Beta-t-EGARCH and level shift process so that the model is able to capture the multiple regime for volatility. However, how the RLS of the Qu and Perron model nest the Beta-t-EGARCH one? Fortunately, both models features similar structures. A first issue comes from the normality assumption in the SV model, however the level shifts and the multiple regimes estimated follows an independent process of the short term component. A shock to this process has a permanent effect until the next structural break as argued in QP. Thus, we may think that a t-Student model will not have much effect in the quantity and magnitude of the multiple regimen detected.

Hence, I add the volatility process estimated from the stochastic volatility framework as an independent process of the score in order to understand the consequences of the level shift process in the persistence of the process under the DCS setting, which has no yet analyzed.

The model I propose to model the scale with RLS for financial returns is as follow:

$$\begin{aligned} y_t &= \exp(\lambda_{t|t-1} + \gamma\mu_t)\varepsilon_t, \\ \lambda_{t+1|t} &= \omega(1-\phi) + \phi\lambda_{t|t-1} + \kappa u_t, \\ \mu_{t+1} &= \mu_t + \sigma_\eta \delta_t \eta_t. \end{aligned} \tag{6}$$

Where $\lambda_{t|t-1}$ represents the scale of a t-Student distribution u_t is proportional to the score of the logarithm of likelihood of a t-Student distribution One of the advantages of the Beta-t-EGARCH related to other models such as GARCH is that it models directly the scale parameter of a t-Student distribution. $(1-\phi)\omega$ represents the drift of this process, and ϕ the short memory parameter, u_t are the scores obtained from the t-distribution.

3.3 Modelling Asymmetries

Because financial markets do not react in the same way to positive than negative shocks, most known as impact curve news. Here I adapt the model proposed above with the features of the leverage model of Nelson (1991) and Taylor (2005), I will follow a similar structure, thus a negative impact on the returns will have a greater effect over the volatility:

$$
\begin{aligned}
y_t &= \exp(\lambda_{t|t-1} + \gamma\mu_t)\varepsilon_t, \\
\lambda_{t+1|t} &= \omega(1-\phi) + \phi\lambda_{t|t-1} + \kappa u_t + \kappa^* sgn(-y_t)(u_t+1), \\
\mu_{t+1} &= \mu_t + \sigma_\eta \delta_t \eta_t.
\end{aligned} \tag{7}
$$

It is excepted that the sign to be positive in the stock market since it is a liquid market and the investor are not much prone to keep their assets in times of high uncertainty and will probably to sell their values in take advantages in a more secure market.

3.4 Estimation Procedure

Estimation of the models in (6) and (7) can be achieved using Maximum likelihood as described in Harvey (2013). Firstly, I independently estimate the RLS process within the SV model of Qu and Perron (2013). Then, I add this information to the Beta-t-EGARCH model in order to model further for multiple jumps in the process of volatility.

Given the demeaned return process, y_t, the parameters of the RLS-Beta-t-EGARCH with asymmetries can be estimated deriving the likelihood with respect of the parameters $\psi = (\omega, \phi, \kappa, \kappa^*, \gamma)$ and the degrees of freedom ν:

$$
\begin{aligned}
\ln L(\psi, \nu) &= T \ln \Gamma\left(\frac{\nu+1}{2}\right) - \frac{T}{2}\ln\pi - T\ln\Gamma(\nu/2) - \frac{T}{2}\ln(\nu) \\
&\quad - \sum_{t=1}^{T} \lambda^{\dagger}_{t|t-1} - \frac{(\nu+1)}{2}\sum_{t=1}^{T} \ln\left(1 + \frac{y_t^2}{ve^{2\lambda^{\dagger}_{t|t-1}}}\right).
\end{aligned}
$$

where $\lambda^{\dagger}_{t|t-1} = \lambda_{t|t-1} + \gamma\mu_t$ The maximization procedure is made using numerical optimization algorithms.

4 Uncertainty Dynamic in the Peruvian Financial Markets

In this section I present the result for the estimation of the Beta-t-EGARCH model with random level shifts. I describe briefly the returns to be used. Hence, I estimate the model in a two-step procedure. Then, I present the main findings about the dynamics of the uncertainty for Peruvian financial markets and how good the proposed model fits the data.

4.1 The Data

I use daily data from January 1998 to December 2016 for both the stock and forex returns. The returns are computed as $r_t = [\log(P_t) - \log(P_{t-1})] * 100$, where P_t corresponds to the Lima Stock General Index and the exchange rate at the end of the day. The dataset were obtained from Bloomberg and the Superintendent of Banks, Insurance and Payments (SBS) of Peru respectively.

Some relevant statistics for these series are reported in Table 1. The sample average for these returns is near to zero, which is common feature in these markets which is driven by market arbitrage conditions. It is also an indicator of how volatile are the financial series, particularly the sample volatilities are relatively higher for stock returns. Moreover, these returns evidence asymmetries in its density measured by a skewness of -0.38. The kurtosis for both returns are high and more than 10. This evidence the presence of extreme observations which are common in these kind of financial data. As a result, the rejection of normality assumption in this dataset using the Jarque and Bera (1987) statistic is expected.

4.2 Random level shifts smoothed estimates

Figure 2 and 3 shows the posterior densities and correlograms of the parameters ϕ, σ_η, β, p of the SV model for both Peruvian financial markets[5]. I have used 20 000 iterations of the algorithm discarding the first half of them. The almost no correlation in the iterations generated, and the well formed shape of the densities

[5]For the results in this section, I have used generation of random numbers by default given in Oxmetrics 6.0.

suggest an efficient estimation toward the convergence of the true values of the parameters in this model. Nonetheless, the results reported here robust to several rational specifications of priors since it has been assumed quite general priors and even diffuse prior for some parameters.

For stock returns and market returns the posterior mean for the size of the shifts, σ_η, are similar for both series with a posterior mean of 1.68 and 1.66 respectively. However, the main difference in both estimation comes from the number of level shifts, for the stock market it ranges from 8 to 19, meanwhile for the forex market it is rather more frequent going from 15 to 60 shifts. It is reflected for the estimates of the Bernoulli parameter, for stock market it is $\hat{p} = 0.003$ thus it means that a level shift occurs each 313 days in average, alike, the stock market has a posterior mean estimate of 0.001, which implies that a shift happens less infrequently, each 883 days in average.

Figures 4 and 5 plots the level shifts process and also their smoothed probabilities of occurrence. The biggest change in regime has probabilities close to one and this constitutes one of the main advantages in this modelling. Standard Markov switching models are only able to capture transitions between two states. RLS brings enough flexibility for a richer analysis.

Interestingly, the main upward shifts in both series are associated to main external and/or internal shock which affected the Peruvian economy and its financial markets. It can be distinguished clearly a period of high instability during the ending of 1990s due to the Asian crisis which affected severely the terms of trade because Peru at that time was mainly a commodities exporter country. Also, both markets, by the end of 2005, display a level shift as the markets reacted to the favorite candidate Ollanta Humala who was winning the main ballot surveys, and who at that stage had offered radical policy proposals and reforms, thus main investors stopped their investments in the country, selling their positions in the stock market and also leaving less dollars in the economy. These events generated relatively more uncertainty in the stock market, which held high levels of uncertainty until the beginning of 2008. In contrast, the forex market whose volatility is in some extent regulated for the Central Bank Peru, shows a regime of relatively stability after around five months.

A second main event that explains a new high regime of uncertainty was the external shock suffered from the global financial crisis. Around June 2008 the stock market saw unprecedent levels of uncertainty lasting for around one year. It is after the reaction of the Peru's government, through a sequence of economic restructuration packages, that the markets could mitigate the considerable effects of the US crisis. Again, due to the quick response of the Central Bank to maintain stable level of volatility, this new regime of uncertainty was steeply controlled, and in 2010 it shown an stable regime of low volatility with levels viewed, before the crisis, in 2006. Not surprisingly for the forex market, after regimes of high volatility it reveals periods of relatively stability due to the action of the monetary authority.

4.3 Fitting RLS-Beta-t-EGARCH models

Table 4 summarize the estimates for the parameters in the Beta-t-EGARCH model which does not account for level shifts. I focus the discussion particularly on the parameter of persistence and the t-Student degrees of freedom. Both series presents high persistence for the stock market it is 0.96, this implies a half life[6] of a shock of 18 days. On the other hand, the forex market has a persistence of 0.97, therefore, a shock to volatility have a half life of 22 days. With respect for the degrees of freedom, the estimates have a single digit. This means that a big shock will have less impact over the volatility than a model which assumes normality, hence its importance in modelling financial series which present high level of kurtosis and heavy tails.

Estimates for the model RLS-Beta-t-EGARCH, which adds random level shifts as an independent explanatory variable in the scale dynamics, is also reported in Table 4 for both series. We can see how the estimates for the persistence parameter reduces to 0.912 and 0.855 for the stock and forex returns respectively. Thus, the half life of a shock reduces to 7 and 4 days. This process can be understood as a process with a short memory where shocks dissipate quicker. The coefficient for γ associated to the RLS component is estimated at 0.05 for the stock volatilities and 0.1, for both returns the estimate results statistically significant with 95 per cent confidence. This results reinforce my hypothesis that the RLS process is a key component for the modelling of the volatility.

[6] Half life is defined as $\ln(2)/\ln(\phi)$.

The third last lines of the Tables provides various measurers of adjustment of the model to the data such as the log likelihood of the model and two information criteria. According to the values of the log likelihood, there is a gain in the fit to the data, when RLS are incorporated. We may find that because of adding a new independent process the RLS provides automatically a better adjust. For this reason, I consider both the Akaike (1973) and the Schwarz (1978) information criteria, denoted as AIC and BIC respectively. Both criteria penalize for an increasing number of the parameters considered in the model. Thus, a lower value from this criteria indicates a better model adjusted by its complexity. Consider these measures, we can see a significant improvement in the fitness of the data when using the model proposed in this dissertation.

For the RLS-Beta-t-EGARCH model with leverage I find for the stock market an expected positive sign leverage with an estimate of 0.014. Indeed, bad news exacerbate the stock market and it generates a much greater volatility than the intrinsic impact of the shock. This is a common characteristic of considerable liquid markets. On the other hand, we may interpret positive shocks in the forex market as a kind of depreciative shocks because it rise the level of the exchange rate, which at the same time generates a loss in value for the national currency. Similarly, a negative shock implies an appreciation shock of the local versus the foreign currency. I find an estimate of -0.022 for the Peruvian forex market. In line with this estimate we can infer that depreciations (positive) shocks generates more uncertainty than the appreciation (negative) shocks. These estimates are consistent with the results for an SV with asymmetries model in Alanya and Rodríguez (2016). The negative coefficient indicates that Central Bank has to care more in the volatility generating from depreciation shocks in its policy to smooth the path of the volatility in this market. A higher volatility from depreciation can be explained due to the vulnerability of a small open economy from speculative currency investors. These new features contribute to the even better fitness of the data evidenced for a lower AIC and BIC criteria.

Note that it is not enough to check the adjustment criteria to argue for the validity of a model. A more objective look how well is the model proposed comes from the analysis of residuals, this should display similarities with the assumption for the error term of the model. To do so, I compute the residuals $\hat{\varepsilon}_t$ from the RLS-Beta-t-EGARCH model with leverage so that $\hat{\varepsilon}_t = y_t \exp(-\hat{\lambda}_{t|t-1} - \hat{\gamma}\hat{\mu}_t)$. Then, if the model is appropriate, the residuals $\hat{\varepsilon}_t$ should display similar properties as the error term ε_t which follow a standardized t-Student distribution. Top panels in Figure 7 and 8 shows the kernel densities of the residuals and the assumed distribution. Despite slightly difference in the peak of the distributions, the residuals can reproduce almost all the moments of the error term. It is important also consider if the residuals are serially correlated. I consider two transformation of the residuals, the logarithm of the autocorrelations and the absolute values, in order to check for robustness. From the median and lower bottom of the same Figures, we can see the sample autocorrelations for both series at all lags, no problems of serial correlation can be appreciated. This evidence how well the RLS-Beta-t-EGARCH with asymmetries model the uncertainty of the Peruvian financial markets.

The plot of volatilities from this model are shown Figure 6. Although controlled for multiple regimes, it can be distinguished episodes of relatively higher uncertainty. For example, between 2008 and 2010, period associated with the financial crisis. It is interesting to see how responsive the stock market is during and before a national election for the presidency in mid 2006 and mid 2011.

5 Monte Carlo Simulations

In order to validate the robustness of our results through this section I study artificial series generated by a data generating process following the RLS-Beta-t-EGARCH model. In the first subsection I briefly summarizes how I conduct the Monte Carlo simulations and in the following subsection I cover the time and spectral properties of the original and simulated series.

5.1 The experimental design

These experiments will be based on 1000 artificial series created from the RLS-Beta-t-EGARCH model with asymmetries in (12) since it is the model that better fit the data. First, for the level shifts process, I iteratively generates the observations for $t = 2, 3, ..., T$ given the value for $t = 1$ being its posterior mean with the estimates obtained in the preceding section so that I generate the simulated random level shift sequences according to:

$$\begin{aligned}\tilde{\mu}_{t+1} &= \tilde{\mu}_t + \hat{\sigma}_\eta \hat{\delta}_t \eta_t, \\ \hat{\delta}_t &\sim B(1, \hat{p}),\end{aligned}$$

with the circumflex symbol denoting the estimates presented in Table 5 for both Peruvian financial markets. Secondly, using these estimates and those for the computed level shifts component, I generate artificial series from the Beta-t-EGARCH model:

$$\begin{aligned}y_t &= \exp(\lambda_{t|t-1} + \hat{\gamma}\tilde{\mu}_t)\varepsilon_t, && (8) \\ \lambda_{t+1|t} &= \hat{\omega}(1-\hat{\phi}) + \hat{\phi}\lambda_{t|t-1} + \hat{\kappa}u_t + \hat{\kappa}^* sgn(-y_t)(u_t+1), \\ u_t &\sim beta(1/2, \hat{\nu}/2), && (9) \\ \mu_{t+1} &= \mu_t + \sigma_\eta \delta_t \eta_t.\end{aligned}$$

I will proceed similarly to Perron and Qu (2010) in their analysis of the time and spectral properties of the simulated series.

5.2 Spurious long memory?

Once created the 1000 series I analyze the time domain property computing the sample autocorrelatios and the frequency domain with the log periodogram of Geweke and Porter-Hudak (1983), for each of the series. The latter consists in the estimate of the long memory parameter at frequencies $j = 1, ..., m$. Thus, the jth Fourier frequency is $w_j = \frac{2\pi j}{T}$ and the log-periodogram estimates d for comes from a least square regression , as follow:

$$\log(I_{x,T}(w_j)) = c - 2d\log(2\sin(w_j/2)) + \varepsilon_j.$$

where $I_{x,T}(w_j)$ is the sample periodogram at frequency w_j.. According to Perron and Qu (2010), a log periodogram which exhibit a changing values for $\hat{d}$ may be a signal of a process with no evidence of long memory. In figures 9 and 10 I plot the log periodogram for the logarithm of the squares of the original stock and forex returns.

The three lines graphed from left to the right indicates frequencies at $T^{1/3}$, $T^{1/2}$ and $T^{2/3.}$ From the empirical evidence found in Perron and Qu (2010), it can be appreciated that for the interval $m = [10, T^{1/3}]$ both periodograms experiment a sudden decline in the value of the estimate $\hat{d}$ by approximately 0.2 points, going from 0.5 to 0.3 in the stock market and in a similar range for the forex market. From frequencies $T^{1/3}$ to $T^{1/2}$ it can be seen a steadily decline in the long memory parameter, although the decrease is more pronounced in the forex market where $\hat{d}$ decays by a level of 0.3. Further, starting at the frequency $T^{2/3.}$ both series display an slow decrease in $\hat{d}$, though with values below the 0.5 indicating that the process starts to be driven by a short memory rather than a long memory process.

All these features according to Perron and Qu (2010) characterize a data generating process consisting in a short memory process plus level shifts leaving less evidence for models who directly tackles the long memory. Additionally, I compute the sample autocorrelations at all lags it also feature patters of short memory process with a quick decline to negative values, for then converge to the zero value. Top panels in Figures 11 and 12 are consistent with this description, being the forex market more erratic than the stock market. These findings are consistent with the estimates reported in Herrera and Rodríguez (2014) who also analyzes these properties, but considering different samples for both series. This is an indicative of the robustness of the results reported in this section.

Both, the autocorrelation function (ACF) and the log periodograms are calculated for each of the 1000 artificial series. Then, simply are taken the averages from these results. The bottom panels in Figures 9 and 11 shows the ACF and the log periodograms simulations for the stock market, meanwhile Figures 10 and 12 for the exchange rate market. At all frequencies from $T^{1/2}$ intervals the simulated series follows the d slow decay. Although at frequencies before $T^{1/2}$ the simulated series does not mimic precisely the patterns observed originally, however they report values in the same range of values starting from 0.4 which is the main feature for described the process as a one of short memory and level shifts. Furthermore, the

ACF follows, in a smoothed way, the trajectory of the original series.

6 Conclusions

This dissertation provides a new approach to deal with series of high persistence commonly treated in the literature as series with long memory. I analyze a Dynamic Conditional Score model for modelling time varying volatilities with random level shifts, the RLS-Beta-t-EGARCH model. This model in contrast to long memory models assume that RLS plays a significant role for the dynamics of the volatility, particularly affecting its persistence. The RLS component consist in a probabilistic multiple change regimen.

In order to estimate the model, I proceed with two steps. Firstly, I estimate the RLS component from the SV model of Qu and Perron (2013) using Bayesian algorithm to estimate the parameters involved in the model, and particle filter techniques in order to estimate the latent processes. Secondly, this component enter as an independent process into the Beta-t-EGARCH model. The next step involve the estimation by maximum likelihood of the RLS-Beta-t-EGARCH model parameters.

The present study shows that the major multiple regime changes estimated are linked to main event that disrupted the Peruvian economy such as a disputed national electoral process, international crisis as the Asian crisis at the beginnings of 1998 and the more recent US financial crisis in 2007. It is noticeable the effect over the estimate of the persistence in volatilities when random level shifts are included in the model reducing the impact of a shock significantly. However, the permanent effects are somewhat controlled by the RLS component. I also modify the model to include asymmetries, the main policy recommendation comes from the estimates for the forex market that reveals that depreciation shocks generates more volatility, and thus, the Central Bank of Peru, which target for a smooth path of the exchange rate volatility, should be more active (intervening the market) during these episodes.

A look to different transforms of the residuals such as the log squared and absolute value reveals no correlation, this means that the model is able to fit well the volatility of the series. A comparison between the density shape of the residuals and the assumed standardized t-Student distribution of the model implies the model capture well different statistical moments such as the skewness and kurtosis, the latter being of significant importance in the analysis of the financial series.

This dissertation opens up several avenues for further research. Both estimated volatilities seems to be correlated as they react with major level upshifts and regime when they face an external shock of consideration. Thus, it would be worthy develop an extension for multivariate multiple regimen and be able to identify common features and co-movements in times of high uncertainty. In addition, it would be desirable to estimate a model in a unique step procedure mixing unobserved component models and parameter driven models as in Bazzi et al. (2017). I am currently researching this approach of estimation instead of the indirect two step estimation adopted here. Further, it may be interesting the modelling of the location at the same time as the scale of the t-Student distribution and may take advantage from a more dynamic and richer model.

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7 Annex

7.1 Tables

Table 1. Descriptive Statistics

Values	Stock Returns	Forex Returns
Mean	0.035	0.005
Median	0.034	0.000
Maximum	12.816	2.209
Minimum	-13.291	-2.304
Standard Deviation	1.381	0.248
Skewness	-0.377	0.030
Kurtosis	13.946	13.301
Jarque-Bera	23019.87	20234.08
Observations	4589	4576

Table 2. Values for the mixing distribution $\varepsilon_t^* \sim \sum_{i=1}^{k} q_i N(m_i, \sigma_i^2)$

i	q_i	m_i	σ_i^2
1	0.0073	-10.12999	5.79596
2	0.10556	-3.97281	2.61369
3	0.00002	-8.56686	5.1795
4	0.04395	2.77786	0.16735
5	0.34001	0.61942	0.64009
6	0.24566	1.79518	0.34023
7	0.2575	-1.08819	1.26261

Table 3. Priors distributions for SV

Parameter	Distribution	Prior mean	Source
ϕ	$\left[\frac{1+\phi}{2}\right]^{19}\left[\frac{1-\phi}{2}\right]^{0.5}$	0.86	Kim et al. (1998)
σ_υ^2	$\mathcal{IG}(2.5, 0.025)$	0.017	Kim et al. (1998)
p	$Beta(1, 40)$	1/41	Qu and Perron (2013)
σ_η^2	$\mathcal{IG}(10, 30)$	3.333	Qu and Perron (2013)
P	$diag(1 \times 10^6, 1 \times 10^6)$	Diffuse	Qu and Perron (2013)

Table 4. Parameter estimates for Beta-t-EGARCH with and without RLS

	Stock Market		Forex Market	
Parameter	Mean	Standard error	Mean	Standard error
Without Random Level Shifts				
ω	-0.094	0.028	-1.951	0.002
ϕ	0.963	0.008	0.970	0.005
κ	0.117	0.011	0.166	0.009
ν	6.213	0.804	4.816	0.164
$\ln L$	-6795.664		1670.020	
AIC	13599.328		-3332.039	
BIC	13625.054		-3306.325	
With Random Level Shifts				
ω	-0.079	0.021	0.363	0.040
ϕ	0.912	0.013	0.855	0.018
κ	0.124	0.009	0.188	0.025
γ	0.049	0.009	0.093	0.047
ν	6.575	0.587	5.155	0.235
$\ln L$	-6768.806		1725.901	
AIC	13547.613		-3441.801	
BIC	13579.770		-3409.658	

Table 5. Parameter estimates for RLS-Beta-t-EGARCH with leverage

	Stock Market		Forex Market	
Parameter	Mean	Standard error	Mean	Standard error
ω	-0.078	0.032	0.313	0.030
ϕ	0.910	0.015	0.855	0.031
κ	0.122	0.011	0.187	0.017
κ^*	0.014	0.010	-0.022	0.012
γ	0.049	0.005	0.091	0.021
ν	6.669	0.348	5.121	0.277
$\ln L$	-6764.039		1732.430	
AIC	13540.078		-3452.859	
BIC	13578.667		-3414.288	

7.2 Figures

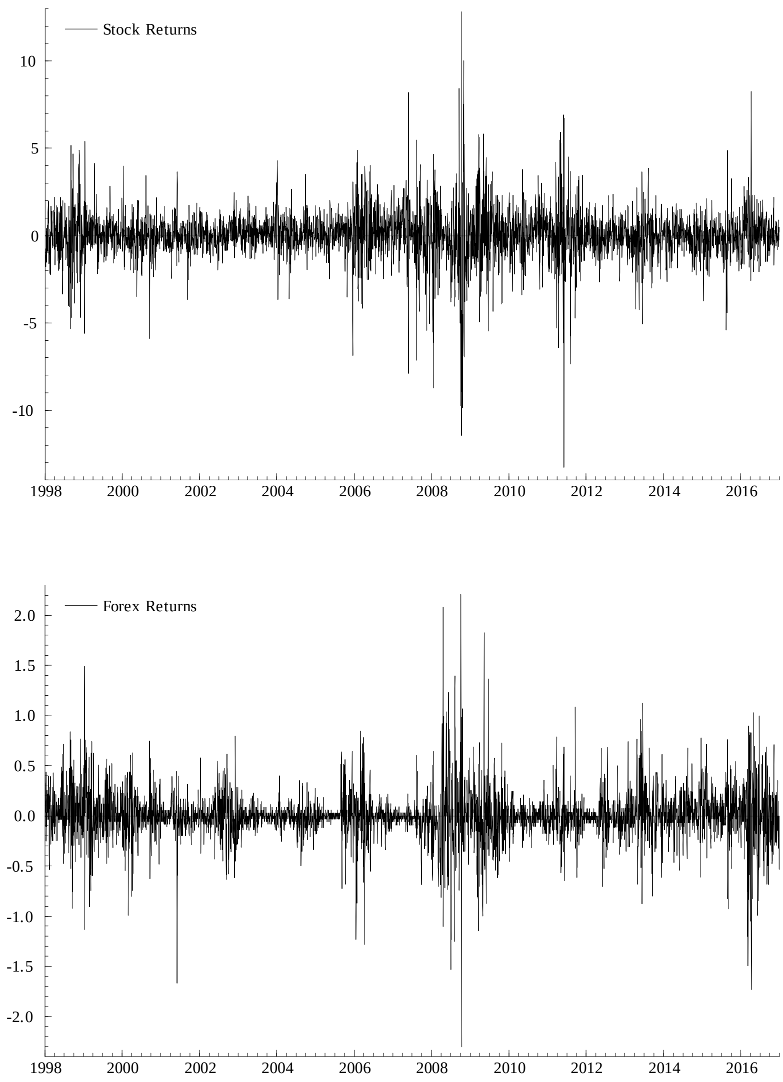

Figure 1. Stock and Forex Returns Series

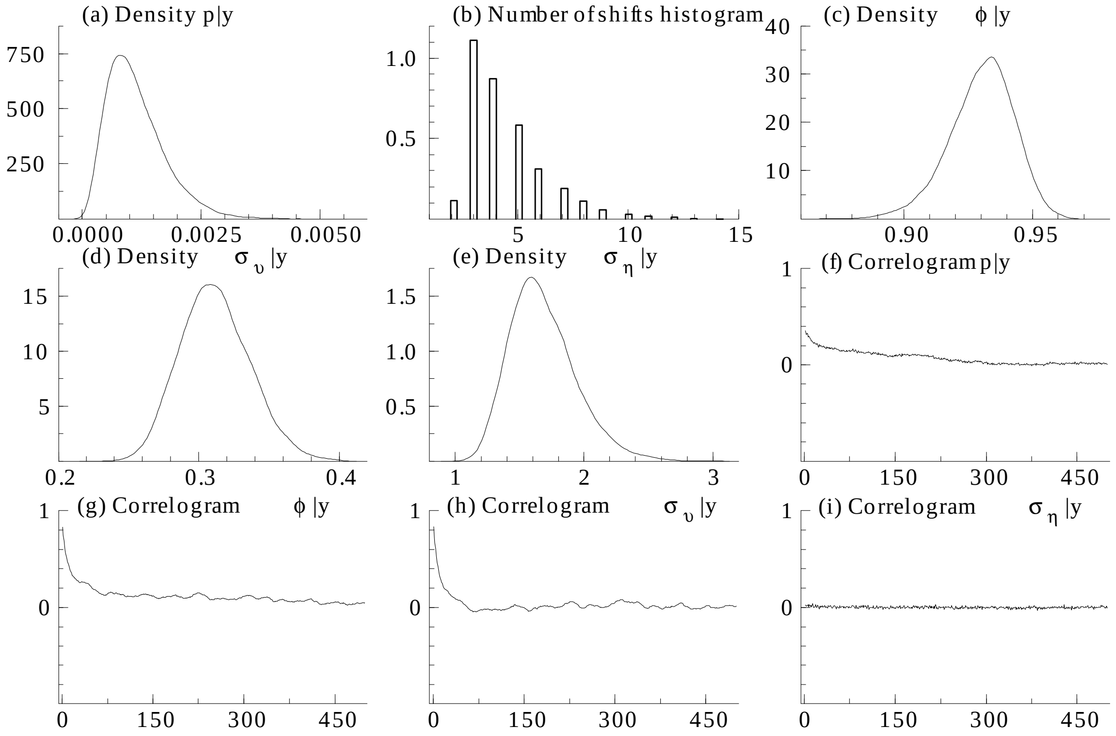

Figure 2. SV model posterior densities and correlograms for Stock returns volatility.

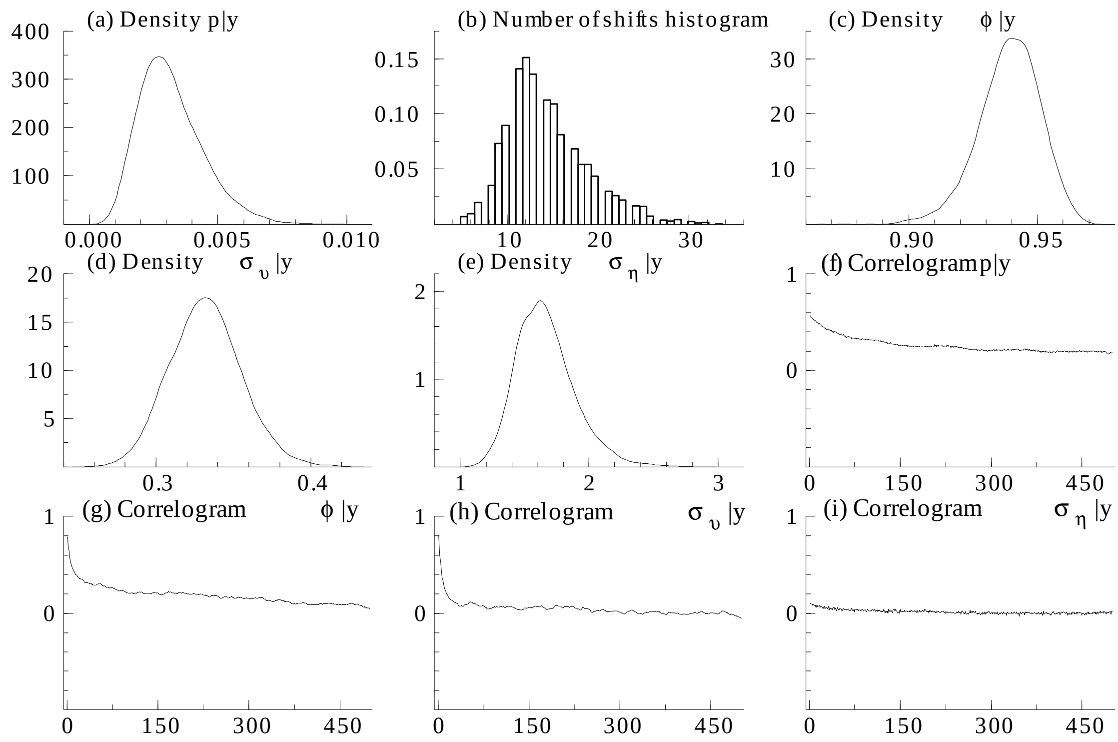

Figure 3. SV model posterior densities and correlograms for Forex returns volatility.

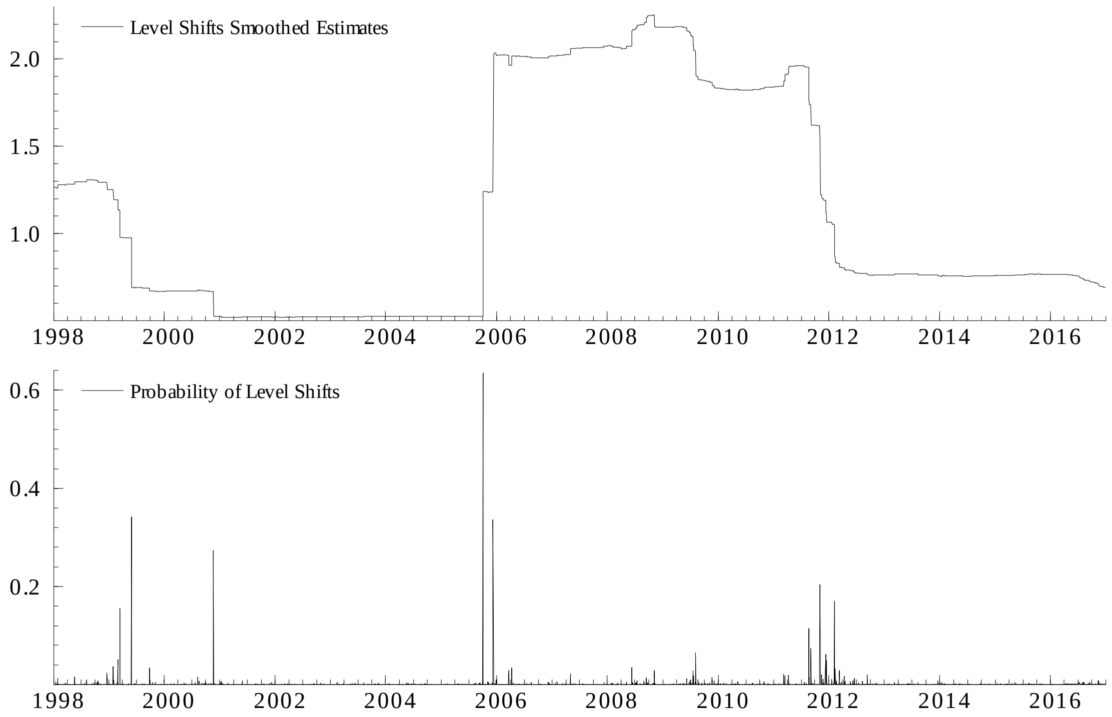

Figure 4. Smoothed estimates of level shifts, $\exp(\mu_t)$, and probabilities of level shifts in the Stock market.

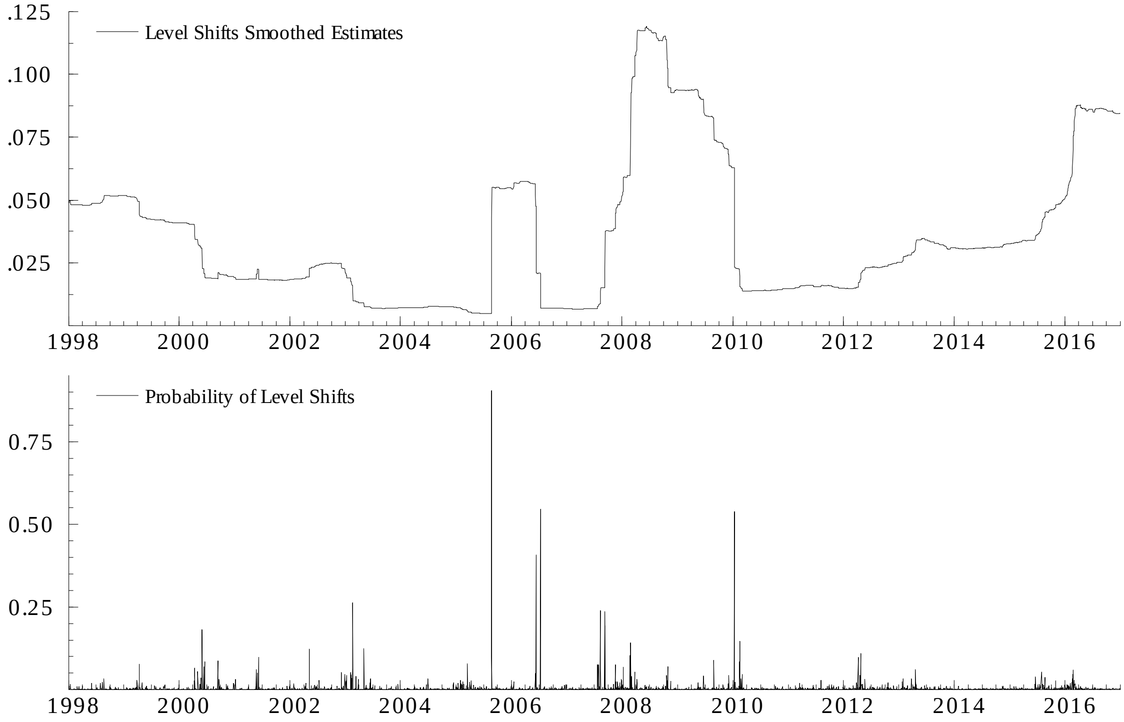

Figure 5. Smoothed estimates of level shifts, $\exp(\mu_t)$, and probabilities of level shifts in the Forex market.

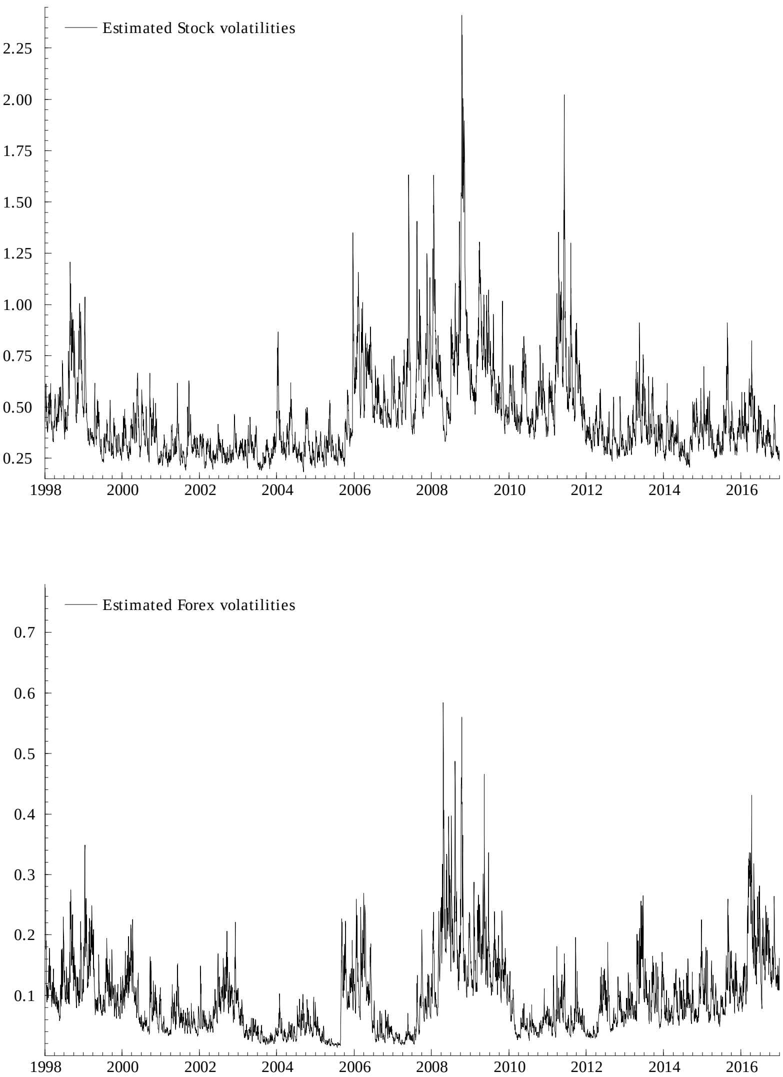

Figure 6. Estimated volatilities for Stock and Forex Returns.

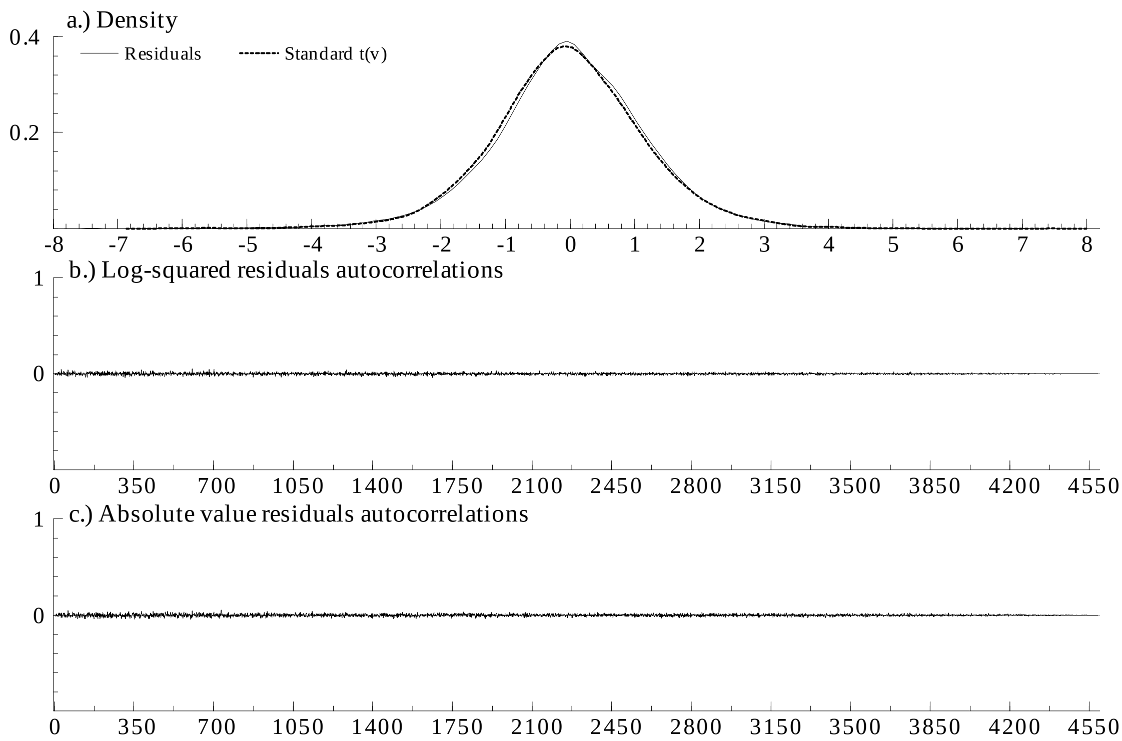

Figure 7. Diagnostic results for Stock market

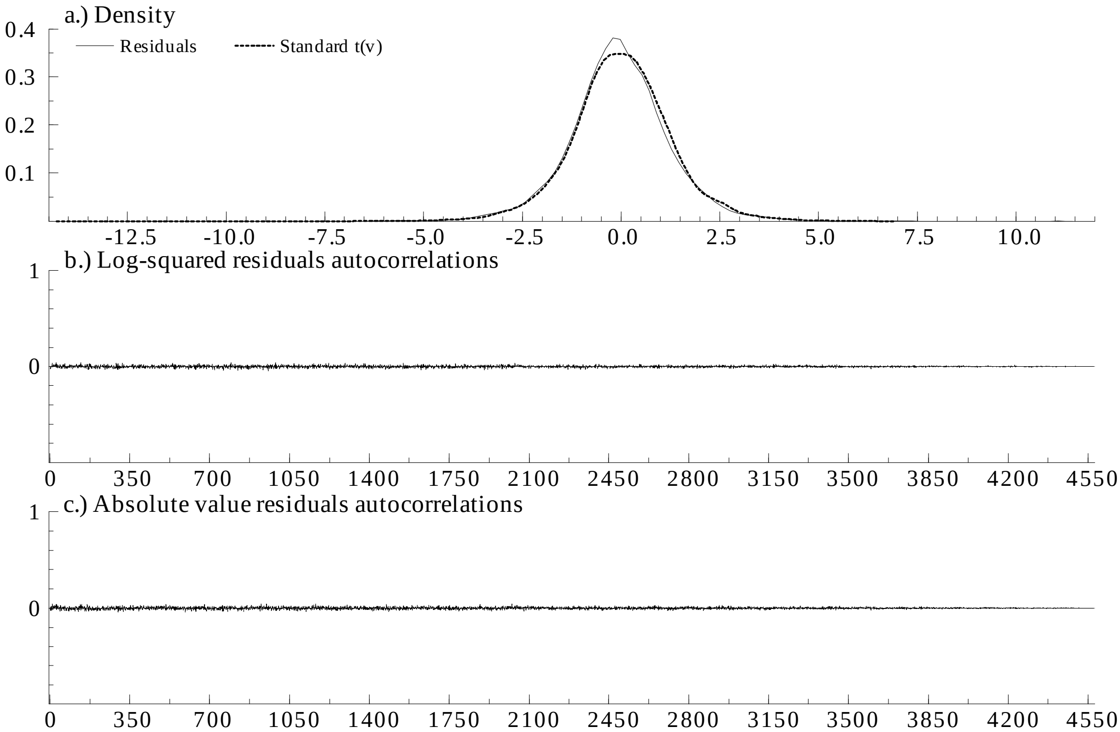

Figure 8. Diagnostic results for Forex market

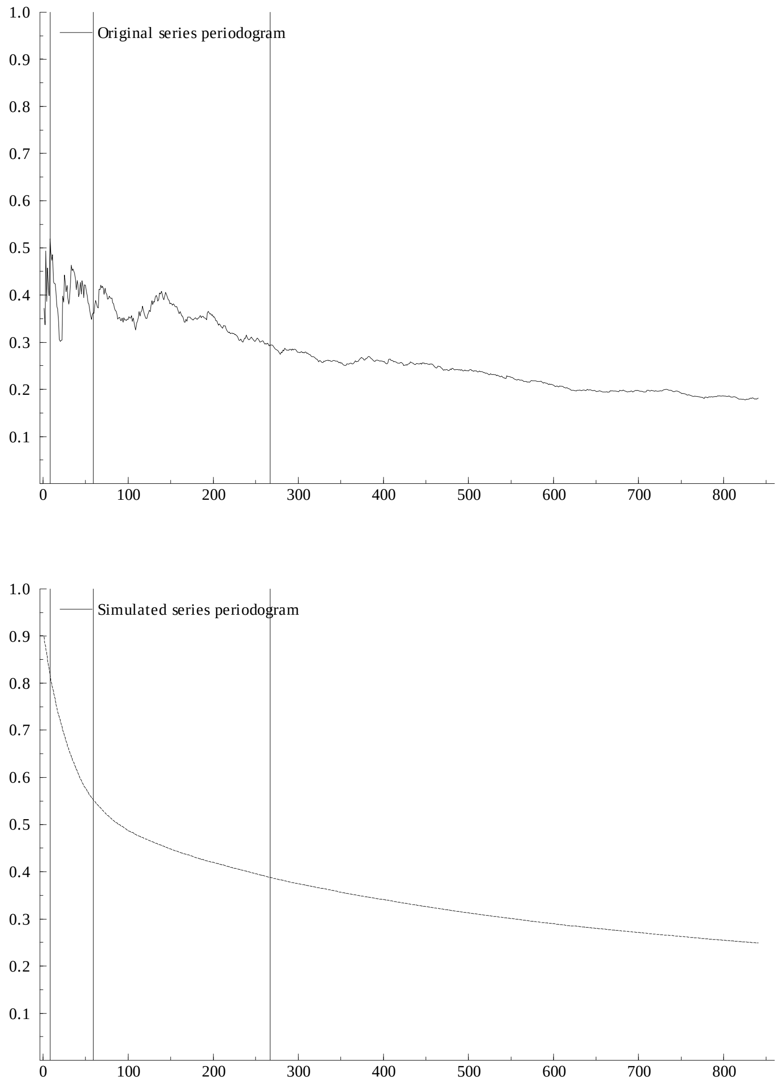

Figure 9. Periodogram for original and simulated Stock market

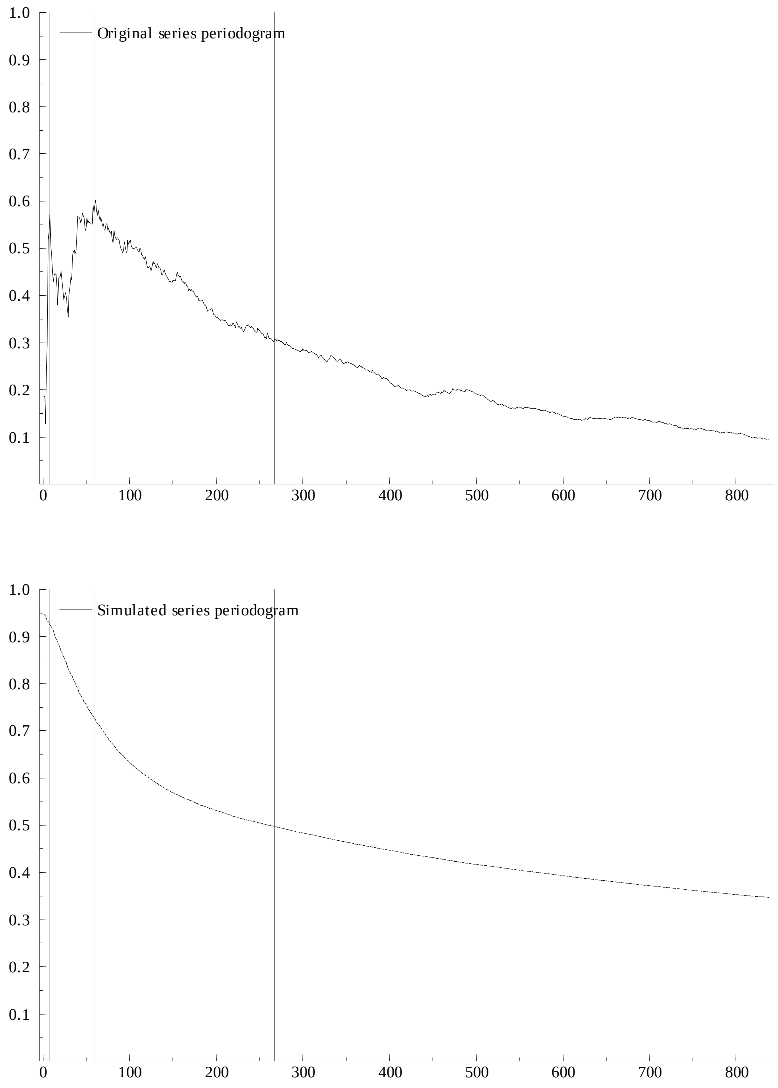

Figure 10. Periodogram for original and simulated Forex market

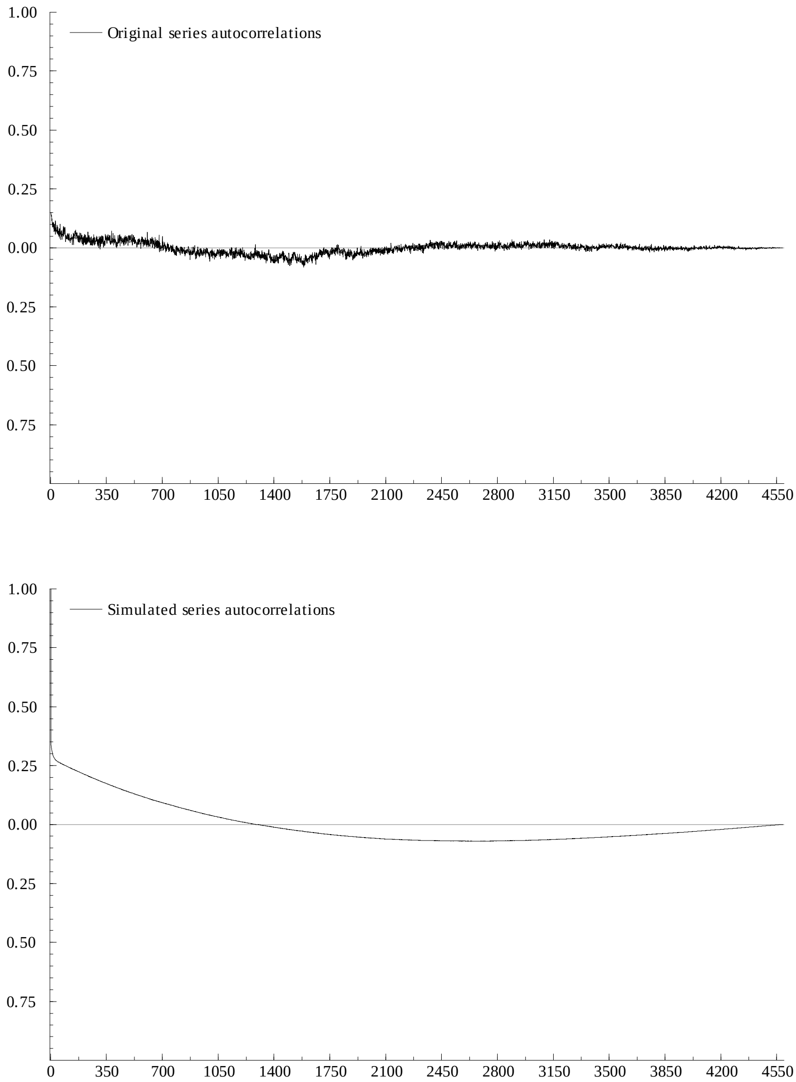

Figure 11. Sample autocorrelations for original and simulated log-squared Stock returns

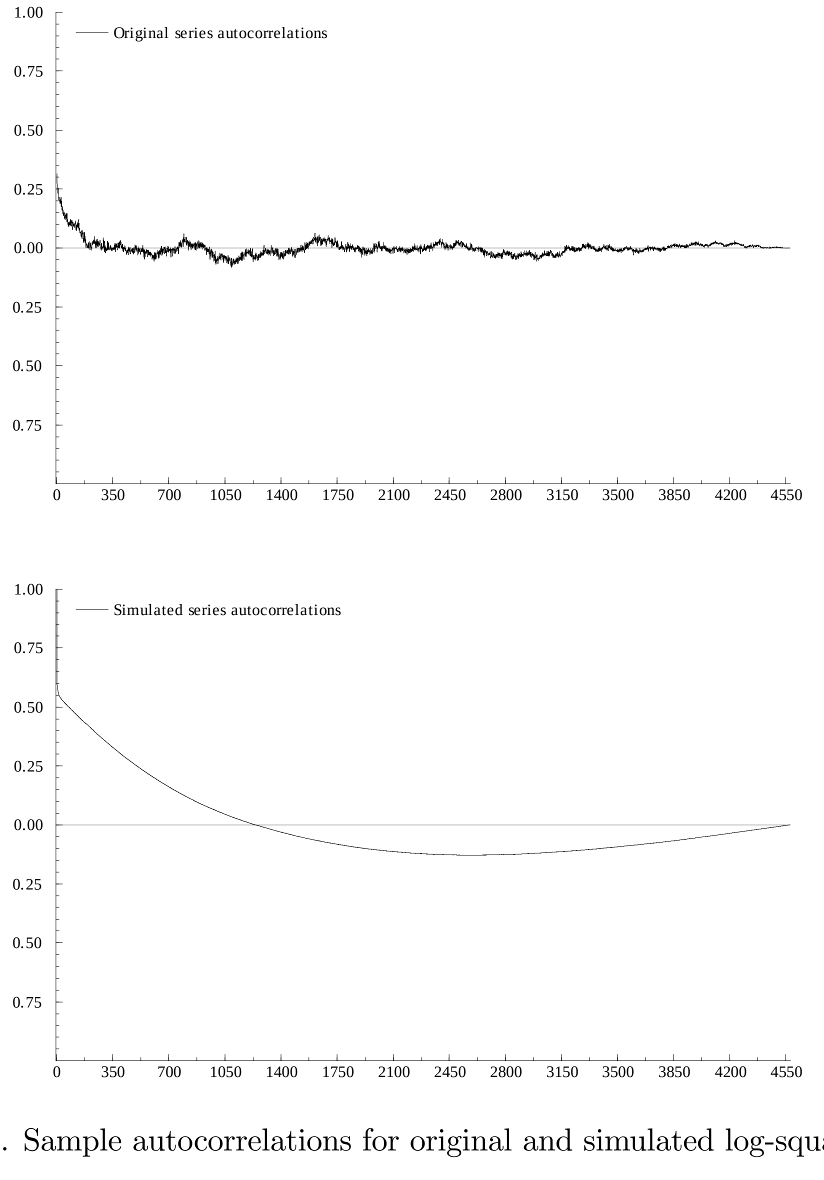

Figure 12. Sample autocorrelations for original and simulated log-squared Forex returns