Additional info by GAS program GasVolaUnivMain

This document gives additional information users should know before implementing the program GasVolaUnivMain.m. The class of volatility models covered by the program is given by

$$
\begin{aligned}
y_t &= \mu + \sigma(f_t)u_t, \qquad u_t \sim p_u(u_t;\theta), \\
f_t &= \omega + \sum_{i=1}^{p} A_i s_{t-i} + \sum_{j=1}^{q} B_j f_{t-j}, \qquad t = 1,\ldots,n,
\end{aligned} \tag{1}
$$

with $p_u(u_t;\theta)$ a standardised disturbance density, $\sigma(f_t)$ a link function and s_t the scaled score. The parameter vector θ is given by

$$
\theta = (\omega, A_1, \ldots, A_p, B_1, \ldots, B_q, \mu, \nu), \tag{2}
$$

and is estimated by the method of maximum likelihood. The parameter which represents the degrees of freedom ν in (2) is estimated only when the standardised disturbance density $p_u(u_t;\theta)$ is Student's t. The user of the program is referred to Creal, Koopman, and Lucas (2013) for more explanation on GAS models.

User input

The user input is located between line 19 and 38 of the program. The following code is copied from the program.

```
19 mdata = xlsread('DJInd19801999.xls');
20 vy = mdata(2:end,5)';
21 dscaling = 1; % Scaling data can improve stability, 1 for no scaling
22 vy = vy.*dscaling;
23 % Distribution: GAUSS, STUD_T
24 idistribution = GAUSS;
25 % Link function: SIGMA (f_t=sigma^2_t), LOG_SIGMA (f_t=log(sigma^2_t))
26 ilinkfunction = LOG_SIGMA;
27 % Scaling score: INV_FISHER, INV_SQRT_FISHER
28 iscalingchoice = INV_FISHER;
29 % Order of GAS model p, q
30 ip = 1; iq = 1;
31 % Standard erros: HESS, SAND
32 istderr = HESS;
33 % Starting values (note the dimensions of ip and iq)
34 domega = 0.01;
35 vA = 0.10; % Extend for higher orders of p, use vector vA = [a ; b ; c ; ..];
36 vB = 0.89; % Extend for higher orders of q, use vector vB = [a ; b ; c ; ..];
B_1 + ... + B_q < 1
37 dmu = 0;
```

```
38 ddf = 5; % Only estimated if idistribution = STUD_T
```

The user input starts by loading the data which needs to be analysed (code line 19). The dataset `DJInd19801999.xls`, available from the same source this document comes from, is loaded as example. In some cases the optimising process becomes more stable if the data is scaled by a factor (code line 21). If the data are returns, a factor of 100 should work well.

1. Choice of disturbance density, (code line 24): available choices are `GAUSS` and `STUD_T`.
2. Choice of link function $\sigma(f_t)$ (code line 26): available choices are `SIGMA` ($f_t = \sigma_t^2$) and `LOG_SIGMA` ($f_t = \log \sigma_t^2$). The `LOG_SIGMA` option is generally more stable.
3. Choice of scaling of the score, (code line 28): available choices are `INV_FISHER` and `INV_SQRT_FISHER`.
4. Order of the GAS model, (code line 30): available choices are any integer > 0 with a maximum dependent on what the data can identify.
5. Choice of standard error type, (code line 32): available choices are `HESS` (empirical Hessian) and `SAND` (sandwich estimator).
6. Starting values for the maximising algorithm, (code line 34 to 38): if the link function is specified as `SIGMA`, the parameter ω is restricted to be $\omega \geq 0$ which is guaranteed by a log transformation of the parameter in the model. No actions for this are required by the user. The user needs to extend the vector of starting values for `vA` (code line 35) and `vB` (code line 36) to the number equal to `s_ip` and `s_iq` (code line 30), respectively. The sum of the elements in `vB` cannot exceed 1. Note that obtaining a global maximum is not always guaranteed and trying different starting values could be useful in some situations.

Computational details

1. Standard errors of the MLE are calculated by inverting the numerically computed Hessian matrix and applying the delta method to the transformed parameter(s).
2. The unconditional mean of f_t is used as initial condition given by $f_0 = \omega(1 - B)^{-1}$.
3. The first `max(s_ip, s_iq)` observations do not contribute directly to the likelihood function as described in, for example, Tsay (2005) p107.

Model output

1. The program output are the BFGS iterations, the maximized log likelihood value and the estimated parameters + standard errors.
2. A figure is plotted with the estimated volatility σ_t in the top panel, the score ∇_t in the mid panel and the scaled score $s_t = S_t \nabla_t$ in the bottom panel, all for $t = 1, \ldots, n$ where S_t is the scaling matrix which depends on the choice of the user.

Example

We illustrate the working of the model with an example. The user input starts at line 249 and ends at line 267. This program comes with a data set of weekly continuously compounded returns from the Dow Jones between 1980 and 1999. The data does not need to be scaled as it won't give any problems with estimating the parameter vector. We start the analysis of the data by selecting the following options

```
s_iDistribution = GAUSS;
s_iLinkFunction = LOG_SIGMA;
s_iScalingChoice = INV_FISHER;
s_ip = 1; s_iq = 1;
s_iStdErr = HESS;
```

and starting values

```
domega = 0;
vA = <0.10>';
vB = <0.89>';
dmu = 0;
```

Note that a starting value for `ddf` does not need to be specified (any number will do). After running the program the output should say

```
fminunc stopped because the size of the current step is less than
the selected value of the step size tolerance.
Log Likelihood value = 2502.14

    'omega'    [-0.9962]    [      0.3319]
    'A1'       [ 0.1008]    [      0.0226]
    'B1'       [ 0.8724]    [      0.0425]
    'mu'       [ 0.0028]    [5.9725e-004]
```

The program should converge in around 27 iterations which takes less than 10 seconds on a modern desktop pc. The maximum likelihood estimate for omega is -0.9962. A negative value is allowed because we selected the link function `LOG_SIGMA` which guarantees positive values for the estimated volatility. The output window should be like the one showed in Figure 1. Next, we change the distribution to the Student's t distribution. For this we change the input block to

```
s_iDistribution = STUD_T;
s_iLinkFunction = LOG_SIGMA;
s_iScalingChoice = INV_FISHER;
s_ip = 1; s_iq = 1;
s_iStdErr = HESS;
```

and starting values

```
domega = 0;
vA = <0.10>';
vB = <0.89>';
dmu = 0;
ddf = 5;
```

After running the program the output should now say

```
fminunc stopped because the size of the current step is less than
the selected value of the step size tolerance.
Log Likelihood value = 2530.95

    'omega'    [-0.2746]    [      0.1330]
    'A1'       [ 0.0661]    [      0.0158]
    'B1'       [ 0.9648]    [      0.0170]
    'mu'       [ 0.0032]    [5.6747e-004]
    'df'       [ 7.3430]    [      1.4702]
```

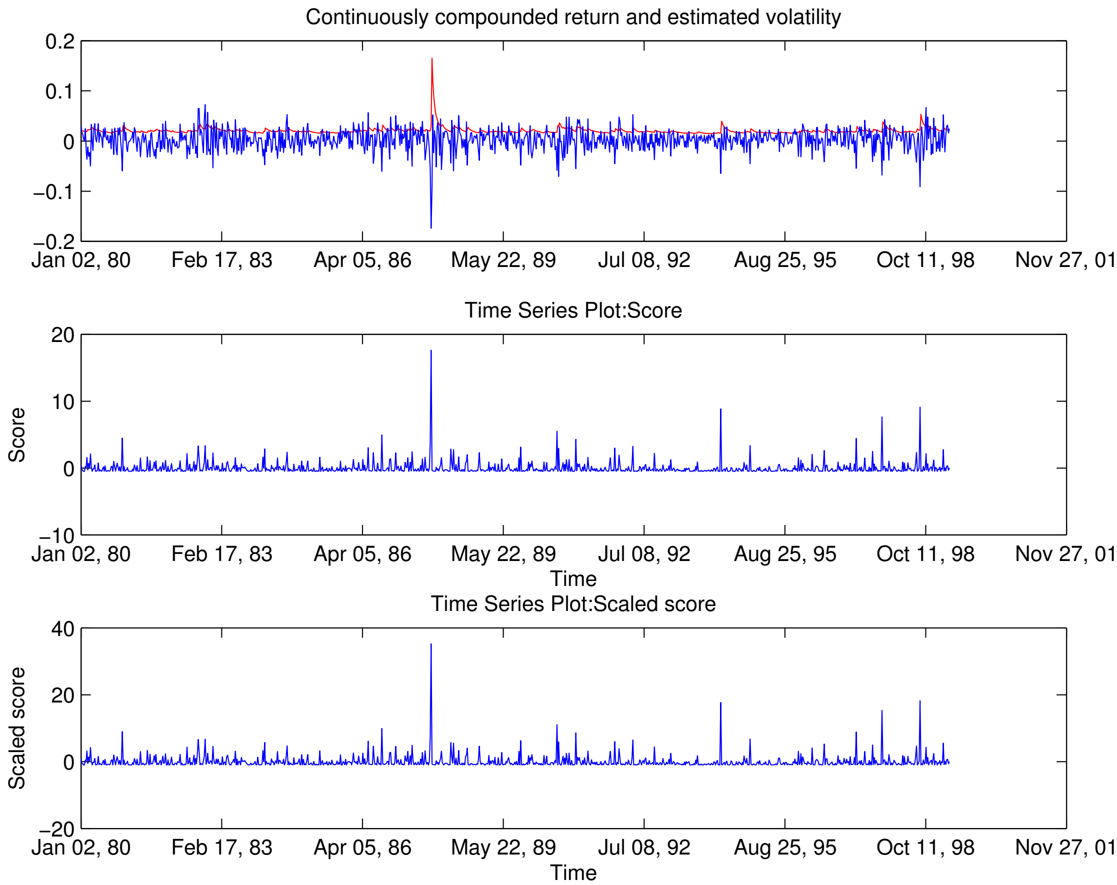
with the output window as showed in Figure 2. As can be seen from Figure 2, the reaction of the model to the Black Monday crash is very different compared to the Gaussian model.

References

Creal, D. D., S. J. Koopman, and A. Lucas (2013). Generalized autoregressive score models with applications. *J. Applied Econometrics 28*, forthcoming.

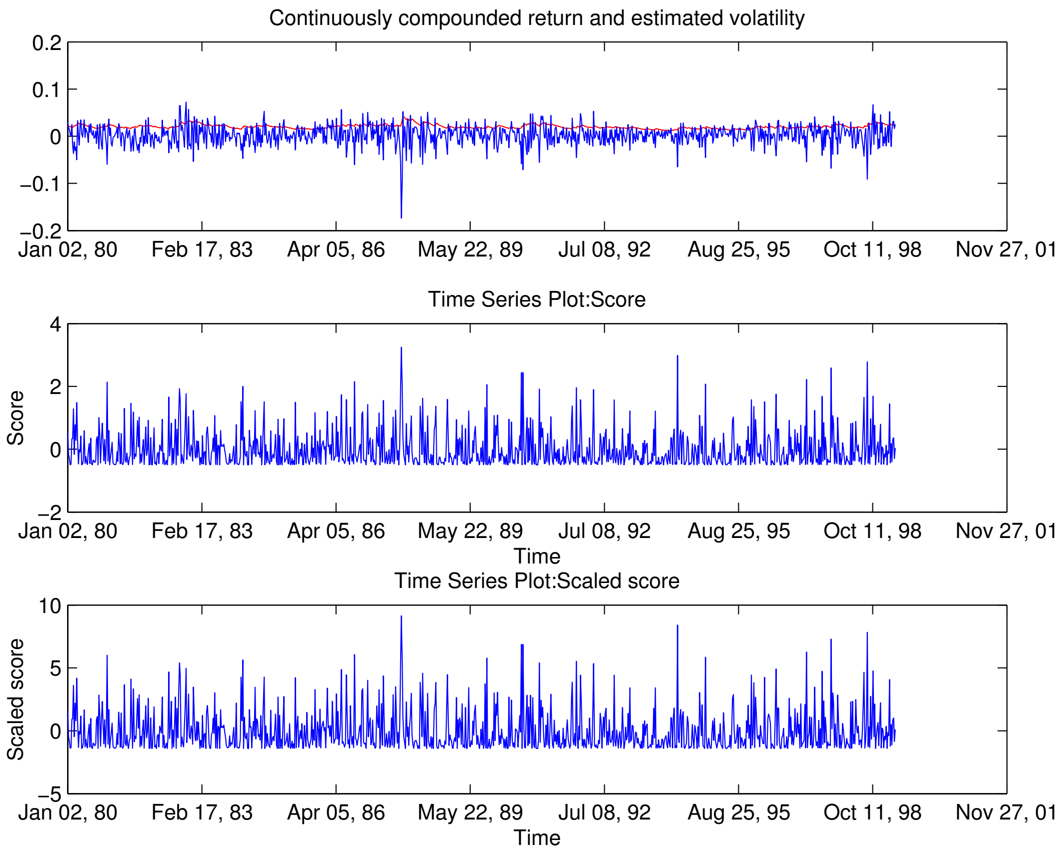
Tsay, R. S. (2005). *Analysis of financial time series* (2nd ed.). New Jersey: Wiley-Interscience.

Figure 1: Gaussian: estimated volatility, score and scaled score



The top panel shows the weekly continuously compounded return from the Dow Jones between 1980 and 1999 and the estimated volatility. Note the big spike in volatility caused by the Black Monday crash of October 19, 1987. The mid panel shows the score and the bottom panel shows the scaled score.

Figure 2: Student t: estimated volatility, score and scaled score



The top panel shows the weekly continuously compounded return from the Dow Jones between 1980 and 1999 and the estimated volatility. The mid panel shows the score and the bottom panel shows the scaled score.