

# 1 Commentary by GAS program GasVolaUniv.ox

This document gives additional information users should know before implementing the program GasVolaUniv.ox. The class of volatility models covered by the program is given by

$$\begin{aligned} y_t &= \mu + \sigma(f_t)u_t, & u_t &\sim p_u(u_t; \theta), \\ f_t &= \omega + \sum_{i=1}^p A_i s_{t-i} + \sum_{j=1}^q B_j f_{t-j}, & t &= 1, \dots, n, \end{aligned} \quad (1)$$

with  $p_u(u_t; \theta)$  a standardised disturbance density,  $\sigma(f_t)$  a link function and  $s_t$  the scaled score. The parameter vector  $\theta$  is given by

$$\theta = (\mu, \omega, A_1, \dots, A_p, B_1, \dots, B_q),$$

and is estimated by the method of maximum likelihood. The user of the program is referred to Creal, Koopman, and Lucas (2013) for more explanation on GAS models. Following is a list of attention points divided into categories.

## User input

1. The user input is between the rows of stars in the main function. It starts with loading the data which needs to be analysed. The dataset “DJInd19801999.xls”, available from the same source this document comes, is loaded as example. Note that in some cases the optimising process becomes more stable if the data is scaled by a factor. If the data are returns, a factor of 100 should work well.
2. The available choices for the disturbance density are GAUSS and STUD\_T.
3. The available choices for the link function  $\sigma(f_t)$  are  $f_t = \sigma_t^2$  and  $f_t = \log \sigma_t^2$ , denoted by SIGMA and LOG\_SIGMA respectively. The LOG\_SIGMA option is generally more stable.
4. The choices for the scaled score  $s_t$  are inverse fisher and inverse square root fisher, denoted by INV\_FISHER and INV\_SQRT\_FISHER respectively.
5. Specify the order of the GAS model by altering “s\_ip” and “s\_iq”
6. The starting values for the maximising algorithm need to be specified by the user. If the link function is specified as SIGMA, the parameter  $\omega$  is restricted to be  $\omega \geq 0$  which is guaranteed by a log transformation of the parameter in the model. No actions for this are required by the user. The user needs to extend the vector of starting values for vA en vB to the number equal to “s\_ip” and “s\_iq”, respectively. The sum of the elements in vB needs to be  $< 1$ . Note that obtaining a global maximum is not always guaranteed and trying different starting values could be useful in some situations.

7. Two choices of obtaining standard errors are available. The first one is the empirical Hessian method denoted by HESS and the second one is the sandwich estimator denoted by SAND.

## Computational details

1. Standard errors of the MLE are calculated by inverting the numerically computed Hessian matrix and applying the delta method to the transformed parameter(s).
2. The unconditional mean of  $f_t$  is used as initial condition given by  $f_0 = \omega(1 - B)^{-1}$ .
3. The first  $\max(\text{s\_ip}, \text{s\_iq})$  observations do not contribute directly to the likelihood function as described in, for example, Tsay (2005) p107.

## Model output

1. The program output is the estimated volatility  $\sigma_t$  in the top panel, the score  $\nabla_t$  in the mid panel and the scaled score  $s_t = S_t \nabla_t$  in the bottom panel, all for  $t = 1, \dots, n$  where  $S_t$  is the scaling matrix which depends on the choice of the user.
2. Activate the function “CompareGaussStudt” to estimates the parameter vector for the available distributions and plot the estimated volatility for each density in one graph.

## References

- Creal, D. D., S. J. Koopman, and A. Lucas (2013). Generalized autoregressive score models with applications. *J. Applied Econometrics* 28, forthcoming.
- Tsay, R. S. (2005). *Analysis of financial time series* (2nd ed.). New Jersey: Wiley-Interscience.